[This question paper contains 4 printed pages.]

(4)

Your Roll No. 2023

Sr. No. of Question Paper : **4530** **E**

Unique Paper Code : 32351401

Name of the Paper : BMATH408- Partial Differential Equations

Name of the Course : **B.Sc.(H) Mathematics**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All sections are compulsory.
3. Marks of each part are indicated.

Section – I

1. Attempt any two out of the following: [7.5+7.5]

(a) Find the integral surfaces of the equation $u\,u_x + u_y = 1$ for the initial data:

$$x(s,0) = s, y(s,0) = 2s, u(s,0) = s.$$

(b) Apply $\sqrt{u} = v$ and $v(x,y) = f(x) + g(y)$ to solve:

$$x^4\,u_x^2 + y^2\,u_y^2 = 4\,u.$$

(c) Find the solution of the initial–value systems

$$u_t + u\,u_x = e^{-x}v,\ v_t - av_x = 0,$$

with $u(x,0) = x$ and $v(x,0) = e^x$.

Section – II

2. Attempt any one out of the following: [6]

(a) Derive the two- dimensional wave equation of the vibrating membrane

$$u_{tt} = c^2(u_{xx} + u_{yy}) + F,$$

where, $c^2 = T/\rho$, and T is the tensile force per unit length

$F = f/\rho$, and f be the external force, acting on the membrane.

(b) Drive the potential equation $\nabla^2 V = 0$, where ∇^2 is known as Laplace operator.

3. Attempt any two out of the following: [6+6]

(a) Determine the general solution of

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2.$$

(b) Given that the parabolic equation

$$u_{xx} = a\, u_t + b\, u_x + c\, u + f,$$

where the coefficients are constants, by the substitution $u = v\, e^{\frac{1}{2}bx}$ and for the case $c = -(b^2/4)$, show that the given equation is reduced to the heat equation

$$v_{xx} = a\, v_t + g,$$

where $g = f\, e^{-bx/2}$.

(c) Reduce the equation

$$(n-1)^2 u_{xx} - y^{2n}\, u_{yy} = n\, y^{2n-1}\, u_y,$$

to canonical form for $n = 1$ and $n = 2$ if possible and also find their solutions.

Section – III

4. Attempt any three parts out of the following: [7+7+7]

(a) Determine the solution of the given below initial-value problem

$$u_{tt} - c^2 u_{xx} = x, \qquad u(x,0) = 0, \quad u_t(x,0) = 3.$$

(b) Obtain the solution of the initial boundary-value problem

$u_{tt} = 9u_{xx}, \quad 0 < x < \infty, \ t > 0,$

$u(x, 0) = 0, \quad 0 \leq x < \infty,$

$u_t(x, 0) = x^3, \quad 0 \leq x < \infty,$

$u_x(0, t) = 0, \quad t \geq 0.$

(c) Solve:

$u_{tt} = c^2 u_{xx},$

$u(x, t) = f(x)$ on $t = t(x),$

$u(x, t) = g(x)$ on $x + ct = 0,$

where $f(0) = g(0)$.

(d) Determine the solution of the initial boundary-value problem:

$u_{tt} = c^2 u_{xx}, \quad 0 < x < l, \ t > 0$

$u(x, 0) = f(x), \quad 0 \leq x \leq l,$

$u_t(x, 0) = g(x), \quad 0 \leq x \leq l,$

$u(0, t) = 0, \quad u(l, t) = 0, \quad t \geq 0.$

Section -- IV

5. Attempt any three out of the following: [7+7+7]

(a) Determine the solution of the initial boundary value problem:

$u_t = 4\, u_{xx}, \quad 0 < x < 1, \ t > 0$

$u(x, 0) = x^2\,(1 - x), \quad 0 \leq x \leq 1,$

$u(0, t) = 0, \quad u(l, t) = 0, \quad t \geq 0.$

(b) Determine the solution of the initial boundary value problem by the method of separation of variables:

$u_{tt} = c^2 u_{xx}, \quad 0 < x < \pi, \ t > 0$

$u(x, 0) = 0, \quad 0 \leq x \leq \pi,$

$u_t(x, 0) = 8 \sin^2 x, \quad 0 \leq x \leq \pi,$

$u(0, t) = 0, \quad u(\pi, t) = 0, \quad t \geq 0.$

(c) Solve by using method of separation of variables:

$$u_{tt} - u_{xx} = h, \qquad 0 < x < 1, \quad t > 0, \; h \text{ is a constant}$$

$$u(x,0) = x^2, \qquad 0 \le x \le 1,$$

$$u_t(x,0) = 0, \qquad 0 \le x \le 1,$$

$$u(0,t) = 0, \quad u(1,t) = 0, \quad t \ge 0.$$

(d) State and prove the uniqueness of solution of the heat conduction problem.

[This question paper contains 2 printed pages.]

(5)

Your Roll No. 2023

Sr. No. of Question Paper : 4686 E

Unique Paper Code : 32351402

Name of the Paper : Riemann Integration and Series of Functions

Name of the Course : B.Sc. (H) Mathematics

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **two** parts from each question.

1(a) Let f be integrable on [a, b], and suppose g is a function on [a, b] such that $g(x) = f(x)$ except for finitely many x in [a, b]. Show g is integrable and $\int_a^b g = \int_a^b f$ (6)

(b) Show that if f is integrable on [a, b] then f^2 also is integrable on [a, b]. (6)

(c) (i) Let f be a continuous function on [a,b] such that $f(x) \geq 0$ for all $x \in [a,b]$. Show that if $\int_a^b f(x)\,dx = 0$ then $f(x) = 0$ for all $x \in [a,b]$ (3)

(ii) Give an example of function such that $|f|$ is integrable on [0,1] but f is not integrable on [0,1]. Justify it. (3)

2(a) State and prove Fundamental Theorem of Calculus I. (6.5)

(b) State Intermediate Value Theorem for Integrals. Evaluate $\lim_{x\to 0} \frac{1}{x}\int_0^x e^{t^2}\,dt$. (6.5)

(c) Let function $f: [0,1] \to R$ be defined as

$$f(x) = \begin{cases} x^2 & if\ x\ is\ rational \\ 0 & if\ x\ is\ irrational \end{cases}$$

Calculate the upper and lower Darboux integrals for f on the interval [0,1]. Is f integrable on [0.1] ? (6.5)

3(a) Examine the convergence of the improper integral $\int_0^\infty e^{-x}x^{n-1}dx$. (6)

(b) Show that the improper integral $\int_\pi^\infty \frac{Sinx}{x}dx$ is convergent but not absolutely convergent. (6)

(c) Determine the convergence or divergence of the improper integral (6)

(i) $\int_0^1 \frac{dx}{x(\ln x)^2}$

(ii) $\int_1^\infty \frac{x dx}{\sqrt{x^3+x}}$

4(a) Show that the sequence

$$f_n(x) = \frac{nx}{1+nx}, \quad x \in [0,1], \; n \in N$$

converges non-uniformly to an integrable function f on [0,1] such that

$$\lim_{n\to\infty} \int_0^1 f_n(x)dx = \int_0^1 f(x)dx$$ (6.5)

(b) Show that the sequence $\{x^2 e^{-nx}\}$ converges uniformly on $[0, \infty)$. (6.5)

(c) Let $\langle f_n \rangle$ be a sequence of continuous function on $A \subset R$ and suppose that $\langle f_n \rangle$ converges uniformly on A to a function $f: A \longrightarrow R$. Show that f is continuous on A. (6.5)

5(a) Let $f_n(x) = \frac{nx}{1+n^2x^2}$ for $x \geq 0$. Show that sequence $\langle f_n \rangle$ converges non-uniformly on $[0, \infty)$ and converges uniformly on $[a, \infty), a > 0$. (6.5)

(b) State and prove Weierstrass M-test for the uniform Convergence of a series of functions. (6.5)

(c) Show that the series of functions $\sum \frac{1}{n^2+x^2}$, converges uniformly on R to a continuous function. (6.5)

6(a) (i) Find the exact interval of convergence of the power series (3)

$$\sum_{n=0}^{\infty} 2^{-n} x^{3n}$$

(ii) Define $\sin x$ as a power series and find its radius of convergence (3)

(b) Prove that $\sum_{n=1}^{\infty} n^2 x^n = \frac{x(x+1)}{(1-x)^3}$ for $|x| < 1$ and hence evaluate $\frac{\sum_{n=1}^{\infty} n^2(-1)^n}{3^n}$. (6)

(c) Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ have radius of convergence $R > 0$. Then f is differentiable on $(-R, R)$ and

$$f'(x) = \sum_{n=1}^{\infty} n a_n x^{n-1} \quad for \; |x| < R.$$ (6)

[This question paper contains 8 printed pages.]

(6)

Your Roll No. 2023

Sr. No. of Question Paper : 4810

Unique Paper Code : 32351403

Name of the Paper : Ring Theory & Linear Algebra – I

Name of the Course : **B.Sc. [Hons.] Mathematics CBCS (LOCF)**

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1. (a) Find all the zero divisors and units in $\mathbb{Z}_3 \oplus \mathbb{Z}_6$. (6)

(b) Prove that characteristic of an integral domain is 0 or prime number p. (6)

(c) State and prove the Subring test (6)

2. (a) Let R be a commutative ring with unity and let A be an ideal of R then prove that R/A is a field if and only if A is a maximal ideal of R. (6)

(b) Let A and B are two ideals of a commutative ring R with unity and A+B=R then show that $A \cap B = AB$. (6)

(c) If an ideal I of a ring R contains a unit then show that I=R. Hence prove that the only ideals of a field F are {0} and F itself. (6)

3. (a) Find all ring homomorphism from $\mathbb{Z}_6$ to $\mathbb{Z}_{15}$. (6.5)

(b) Let $R = \left\{ \begin{bmatrix} a & b \\ b & a \end{bmatrix} \middle| a, b \in \mathbb{Z} \right\}$ and Φ be the mapping

that takes $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ to a-b. Show that

(i) Φ is a ring homomorphism.

(ii) Determine Ker Φ.

(iii) Show that R/Ker Φ is isomorphic to $\mathbb{Z}$. (6.5)

(c) Using homomorphism, prove that an integer n with decimal representation $a_k\, a_{k-1} \cdots \cdots a_0$ is divisible by 9 iff $a_k + a_{k-1} + \ldots + a_0$ is divisible by 9. (6.5)

4. (a) Let V(F) be the vector space of all real valued function over $\mathbb{R}$.

Let $V_e = \{f \in V \mid f(x) = f(-x)\ \forall\ x \in \mathbb{R})$

and $V_o = \{f \in V \mid f(-x) = -f(x)\ \forall\ x \in \mathbb{R})$

Prove that V_e and V_0 are subspaces of V and $V = V_e \oplus V_o$. (6)

(b) Let V(F) be a vector space and let $S_1 \subseteq S_2 \subseteq V$.

Prove that

(i) If S_1 is linearly dependent then S_2 is linearly dependent

(ii) If S_2 is linearly independent then S_1 is linearly independent (6)

(c) Show that $S = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \right\}$ forms a basis for $M_{2X2}(\mathbb{R})$. (6)

5. (a) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3).$$

Find Null space and Range space of T and verify Dimension Theorem. (6.5)

(b) Define $T: M_{2X2}(\mathbb{R}) \to P_2(\mathbb{R})$ by $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a+b) + (2d)x + bx^2$

Let $\beta = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ and $\gamma = \{1, x, x^2\}$ be basis of $M_{2X2}(\mathbb{R})$ and $P_2(\mathbb{R})$ respectively. Compute $[T]_\beta^\gamma$. (6.5)

(c) Let V and W be vector spaces over F, and suppose that $\{v_1, v_2, \ldots, v_n\}$ be a basis for V. For $w_1, w_2, \ldots, w_n$ in W. Prove that there exists exactly one linear transformation $T: V \to W$ such that $T(v_i) = w_i$ for $i = 1, 2, \ldots, n$. (6.5)

6. (a) Let T be the linear operator on $\mathbb{R}^2$ define by

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a+b \\ a-3b \end{pmatrix}$$

Let β be the standard ordered basis for $\mathbb{R}^2$ and let

$$\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}.$$

Find $[T]_{\beta'}$. (6.5)

(b) Let V and W be finite dimensional vector spaces with ordered basis β and γ respectively. Let $T: V \to W$ be linear. Then T is invertible if and only if $[T]_\beta^\gamma$ is invertible.

Furthermore, $\left[T^{-1}\right]_\gamma^\beta = \left([T]_\beta^\gamma\right)^{-1}$. (6.5)

(c) Let V, W and Z be finite dimensional vector spaces with ordered basis α, β, γ respectively. Let $T: V \rightarrow W$ and $U: W \rightarrow Z$ be linear transformations.

Then $[UT]_{\alpha}^{\gamma} = [U]_{\beta}^{\gamma}[T]_{\alpha}^{\beta}$. (6.5)